Water exchange between the Okhotsk Sea and the North Pacific Ocean estimated by simple models

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Introduction

Water exchange between the Okhotsk Sea and the North Pacific Ocean plays an important role in the formation process of the North Pacific Intermediate Water (e.g., Talley and Nagata, 1995; Yasuda, 1997). However, the detailed mechanism of the exchange has not yet been clarified. We estimate the rate of the water exchange by using two simple models. One model is similar to Minato and Kimura (1980) which is used to estimate the flow rate of the Tsushima Current, and the analytical result is discussed by the Island Rule derived by Godfrey (1989), Pedlosky (1994), and Pedlosky et al. (1997). The other is a hydraulic theory derived by Gill (1977), and this hydraulic theory is applied to the Bussol Strait to estimate the flow rate.

Island rule

We use a single layer of homogeneous fluid over a passive quiescent fluid (one-and-a-half layer model), and the Okhotsk Sea and the North Pacific Ocean are modeled as rectangular basins separated by a vertical thin wall except for two gaps as shown in Figure 1: the northern gap corresponds to the Kruzenshterna Strait and the southern gap to the Bussol Strait. The islands between these two straits are replaced with a single island, and this part will be called "the island" hereafter. The sill depths of the straits are assumed to be deeper than the bottom of the surface layer. Since we are discussing only the volume transport, the size of the sea and ocean is not a determinative factor, except for the integration route of wind stress over the Pacific Ocean.

A similar model was used by Minato and Kimura (1980) to estimate transport between the Japan Sea and the Pacific Ocean. By checking their result, it is found that their estimated transport depends strongly on their treatment of the friction term near the strait. They parameterized the frictional effect by "the effective distance from the channel in which the pressure gradient is dominant". This parameter is not clearly defined, and is hard to estimate from observations. A different parameterization is used in this paper.

By using the rectangular coordinates and taking x eastward and y northward, the horizontal momentum equations of the surface homogeneous layer can be expressed in the form:

$$\frac{\partial \vec{u}}{\partial t} + (\zeta + f)\hat{k} \times \vec{u} = -\nabla \left(g' D + \frac{|\vec{u}|^2}{2} \right) + \mathrm{Diss}(\vec{u}) + \vec{T} \quad (1)$$

where $\vec{u}$ is the horizontal velocity (u, v), ζ the vorticity $(= \partial v/\partial x - \partial u/\partial y)$, f the Coriolis parameter, $\hat{k}$ the upward unit vector, g' the reduced gravity, and D the thickness of the surface layer. $\mathrm{Diss}(\vec{u})$ represents the dissipation term, and $\vec{T}$ the wind stress per unit density divided by the fluid depth. The first term of the right-hand side of (1) disappears by

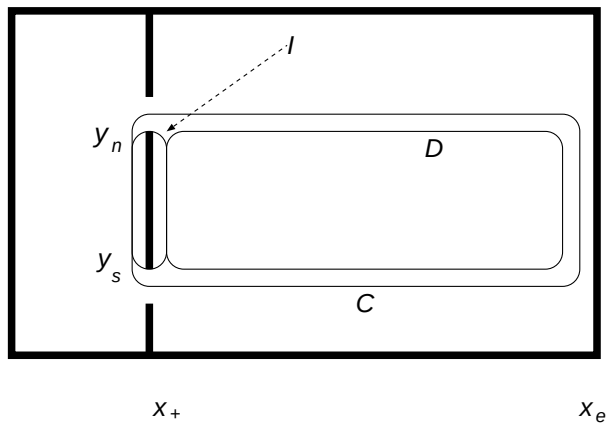

Fig. 1 Schematic view of the model. A square basin is separated by a thin wall with two gaps. The wall represents the Kuril Islands, and the northern gap represents the Kruzenshterna Strait and the southern gap the Bussol Strait. The right larger basin is the Pacific Ocean, and the left smaller basin the Okhotsk Sea. The terms y_n and y_s denote the latitudes of the northern and southern tips of the island, respectively, and x_+ and x_e denote the positions of the western and eastern coasts of the Pacific. Three integration contours used in the text are schematically shown with I, C, and D: the zonal parts of C run at y_n and y_s, and the western meridional part of D and the eastern part of I should be taken just at the eastern side of the island coast (x_+) of I and D.

integrating around the circuit C in Figure 1, and we have

$$\frac{\partial}{\partial t}\oint_C \vec{u}\cdot \mathrm{d}\vec{t} + \oint_C (\zeta + f)\vec{u}\cdot \mathrm{d}\vec{n} = \oint_C \mathrm{Diss}(\vec{u})\cdot \mathrm{d}\vec{t} + \oint_C \vec{T}\cdot \mathrm{d}\vec{t} \quad (2)$$

where $\mathrm{d}\vec{t}$ is the infinitesimal vector along C and $\mathrm{d}\vec{n}$ is the infinitesimal vector normal to C such that $\mathrm{d}\vec{t}\times\hat{k} = \mathrm{d}\vec{n}$. We assume that the flow is steady. Under the quasi-geostrophic assumption, ζ is negligible ($\zeta \ll f$) and the dissipation is also negligible as the circuit C does not reach the western boundary layer. D, which is implicitly included in $\vec{T}$, can be approximated by the surface layer thickness averaged over the whole modeled basin D_0 under the quasi-geostrophic assumption. Multiplying (2) by D_0, we have

$$(f_n - f_s)\Phi = -D_0 \oint_C \vec{T}\cdot \mathrm{d}\vec{t} \quad (3)$$

where Φ is the transport through each channel or the meridional transport between the island and the eastern boundary of the North Pacific, and f_n and f_s are the Coriolis parameters at the latitudes of the north and the south ends of the island, respectively. This corresponds to the Island Rule obtained by Godfrey (1989). We introduce the Sverdrup stream function

$$\beta\Psi_s = -\int_x^{x_e} \hat{k}\cdot\nabla\times D_0\vec{T}\,\mathrm{d}x \quad (4)$$

where x_e is the zonal position of the eastern boundary (Fig. 1). The integral circuit C can be replaced by two circuits, I and D, as shown in Figure 1. The right-hand side of (3) can be written as

$$\oint_C D_0\vec{T}\cdot \mathrm{d}\vec{t} = \left(\oint_I + \oint_D\right) D_0\vec{T}\cdot \mathrm{d}\vec{t} = \oint_I D_0\vec{T}\cdot \mathrm{d}\vec{t} - \beta\int_{y_s}^{y_n} \Psi_s(x_+, y)\,\mathrm{d}y$$

where y_n ($\approx 48°$) and y_s ($\approx 46°$) are the meridional positions of the northern and southern tips of the island, respectively (Fig. 1), and x_+ is the position of the eastern coast of the island. Since the island is thin, the integration in the first term on the right-hand side is much smaller than that of the second term, and (3) becomes

$$\Phi = \frac{1}{y_n - y_s}\int_{y_s}^{y_n} \Psi_s(x_+, y)\mathrm{d}y \quad (5)$$

This is the Island Rule obtained by Pedlosky (1994).

Assuming the Sverdrup balance (4), Ueno (personal communication) calculated the stream function in the North Pacific Ocean (Fig. 2) using the wind field given by Hellerman and Rosenstein (1983). The resulting value of the stream function along the east coast of the island is applied to (5), and the outflow transport through the Bussol Strait is estimated at about 10 Sv (1 Sv = 10^6 m^3 s^{-1}). Ueno also estimated the streamfunction in the North Pacific from the Levitus climatological density field, under several assumptions (the result is not shown here). If we use Ueno's result, the volume transport through the strait becomes 8 Sv if the reference level is taken at 1500 dbar and 10 Sv if the level taken at 2000 dbar.

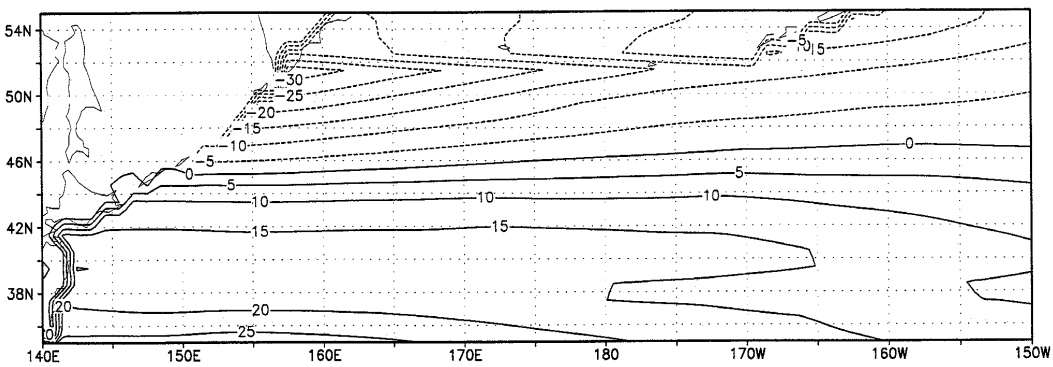

Fig. 2 Stream function (in Sverdrups) in the North Pacific Ocean which is calculated from the Hellerman and Rosenstein (1983) wind field under the assumption of the Sverdrup balance. The contour of the stream function is drawn so that the transport between two adjacent lines indicates 5 Sv, and the value at the east coast is set to zero.

Hydraulic theory

An example of a density cross-section from the Okhotsk Sea to the Pacific Ocean is shown in Figure 3 (Yasuda, 1997). The thickness of the surface layer (see the depth of 27.0 σ_θ) in the Okhotsk Sea is much greater than that in the Pacific Ocean, and a steep slope is seen just at the position of the strait. If the flow direction is from left to right, this sharp slope may represent that the flow state changes from subcritical to supercritical at the narrowest portion of the strait, and the flow may be controlled by hydraulic effect. In the case of the controlled flow and in the case that the flow is vertically uniform (one-layer flow), the volume transport and the Bernoulli function (or energy) are not independent near and upstream of the strait, and the transport can be given by the Bernoulli function and by the topography.

Whitehead et al. (1974) extended the idea of controlled flow in a rotating frame. They considered the case where potential vorticity is zero, and discussed the controlled behavior of a bottom flow. Gill (1977) analyzed a flow with uniform potential vorticity. Here, Gill's model is used to estimate the transport through the Bussol Strait. Since the deformation radius at the strait (roughly 20 km) is comparable to the width of the Strait (roughly 20 km for 1000 m depth), the flow would feel the rotation.

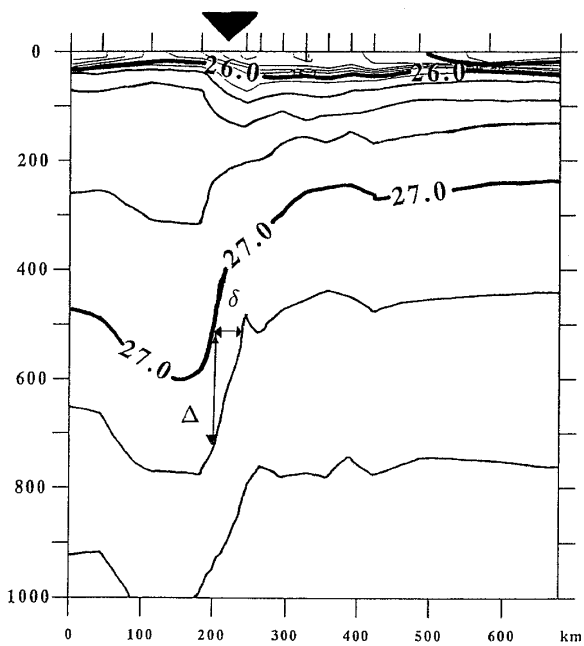

Fig. 3 An example of the density cross-section from the Okhotsk Sea to the Pacific Ocean (Yasuda, 1997). The position of the Kuril Islands is shown by a black triangle above the figure. The depth in meters is taken in the ordinate, and the horizontal distance in kilometers is taken in the abscissa. The values δ and Δ show the horizontal and vertical differences used in the Discussion.

The y-axis is taken in the flow direction (along the center of the strait), and the topography is assumed to be symmetrical to the y-axis (see Figure 4). The x-coordinate is taken horizontally and perpendicular to the y-axis. The right and left coasts are denoted by $x = \pm w(y)$. Starting from the steady inviscid equation of one-layer flow

$$-(f+\zeta)\frac{\psi_x}{D} = -B_x, \tag{6}$$

$$-(f+\zeta)\frac{\psi_y}{D} = -B_y \tag{7}$$

where D is the thickness of the layer and ψ is the stream function:

$$Du = -\psi_x,$$
$$Dv = \psi_y,$$

ζ is the vorticity ($\zeta = v_x - u_y$) and B is the Bernoulli function

$$B = \frac{u^2+v^2}{2} + g'D.$$

Two equations governing the flow of a constant potential vorticity (f/D_∞ where D_∞ is the layer thickness at the center of the far upstream region) under the assumption that flow varies gradually enough ($u \ll v$ and $u_y \ll v_x$). (See Gill (1977)

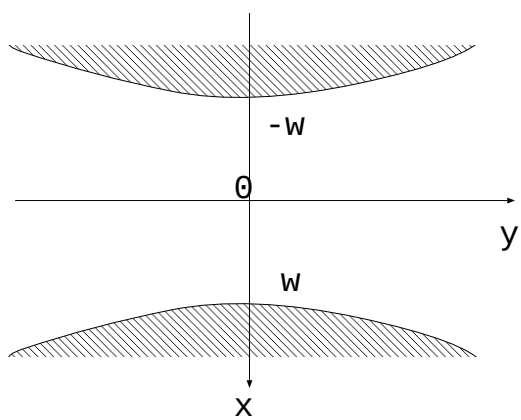

Fig. 4 Schematic view of flow through a strait (Gill's hydraulic model, 1977). The fluid flows from left to right in the y direction; x is taken perpendicular to y. The coasts are set symmetrical to the y-axis. The right coast (looking downstream) is denoted by $x = w(y)$.

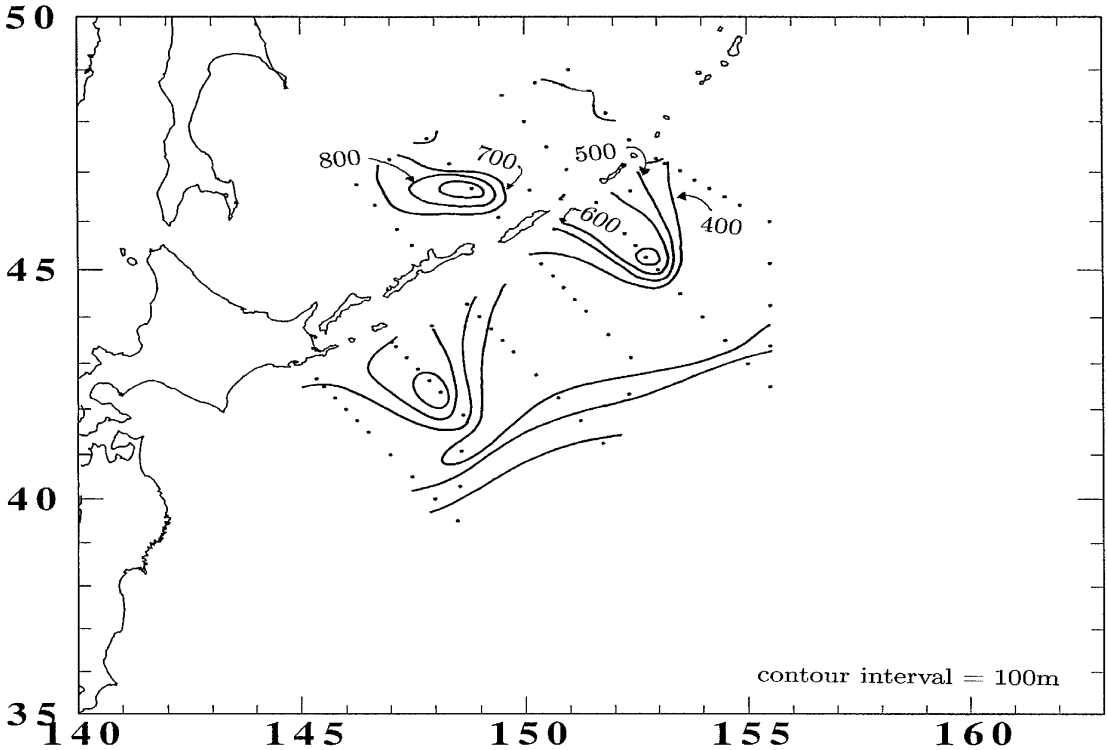

Fig. 5 The depth distribution of the isopycnal surface of $\sigma_\theta = 27.1$ from CTD observation of the R/V *Hokko-maru* in August–September 1991 (Kawasaki and Kono, 1992). Only the contours around the Bussol Strait are numbered. Contour interval is 100 m. Dots show the observation points.

for the detailed derivation.)

$$\delta D \cdot \bar{D} = -\frac{f}{2g}\Phi \quad (8)$$

$$(\bar{D} - D_\infty)^2 t^2 + \delta D^2 t^{-2} + 2D_\infty \bar{D} = \frac{f}{g}\Phi + 2D_\infty B_u/g \quad (9)$$

where $2\delta D$ and $2\bar{D}$ is the difference between and the sum of the water depths at the right (looking downstream) coast and the left coast, respectively, Φ is the transport, and B_u the value of B on the right wall in the far upstream region. The term t is a non-dimensional width and $t = \tanh(wf/\sqrt{g' D_\infty})$. The condition to realize the controlled flow is given by

$$\frac{\partial}{\partial \bar{D}}[\text{Equation (9)}] = 0. \quad (10)$$

If the controlled flow is determined, Φ can be determined by three parameters, t, D_∞, and B_u.

Equation (9) requires that the flow in the upstream region takes the form of boundary jets along the both coasts. The depth D_∞ can be defined in the quiescent region away from the boundary. Denoting the thickness of the layer upstream at the right coast with D_u, the transport (Q_u) of the jet along to the right coast is $g'(D_u^2 - D_\infty^2)/(2f)$, and B_u is given by $B_u = g'(D_u^2 + D_\infty^2)/(2D_\infty)$.

An example of the depth distribution of the isopycnal surface of $\sigma_\theta = 27.1$ is shown in Figure 5 from CTD observations on board the R/V *Hokko-maru* in August–September 1991 (Kawasaki and Kono, 1992). The value of the surface layer thickness in the far upstream region (D_∞) is assumed to be 700 m, from Figure 5. The densities of the surface and bottom layers are 26.8 σ_θ and 27.4 σ_θ, respectively, assumed from the density distribution shown in Figure 3. Then, the reduced gravity g' becomes 5.7 $\times 10^{-3}$ m s^{-2}. The term w is taken as 20 km from the width of the Bussol Strait.

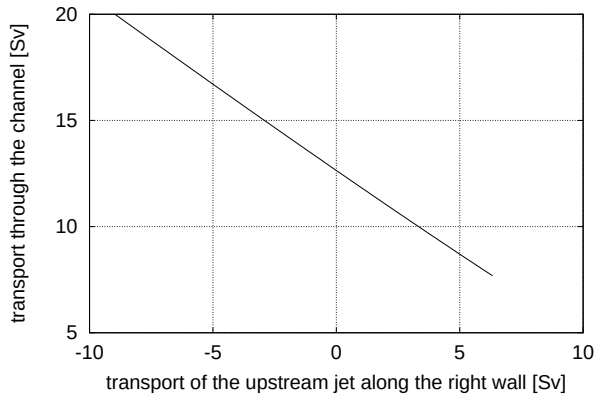

Fig. 6 Transport through the strait as a function of the transport of the upstream jet along the right side of the coast. The estimation is based on Gill's hydraulic model (1977) of constant potential vorticity flow. See the text for details.

The undetermined values are $\bar{D}$, B_u and Φ, and we have two equations (9) and (10). So, we have only one independent variable, and we select the transport of the jet along the right coast (Q_u) as the independent variable. The relation between Q_u and Ψ (the transport through the channel) is shown in Figure 6. If Q_u is positive and very large, the jet separates from the left coast. The curve is not shown beyond this separation point.

The Okhotsk Sea Water is generally flowing out through the Bussol Strait. If Q_u is taken to be positive, the transport through the Bussol Strait is less than 12 Sv and greater than 8 Sv, depending on the strength of the incoming jet. The northeastward jet in the Okhotsk Sea along the Kuril Islands south of the Bussol Strait is not so prominent, and the upstream jet along the right coast may be weak. Then the transport through the strait would be about 10 Sv, although the model would not be applicable directly to the real Bussol Strait.

Discussion

If the flow is always subcritical, hydraulic theory cannot be applied. The Island Rule is basically applicable to subcritical flow. However, the Island Rule does not give any detailed structure near the strait such as the asymmetric density field in the strait. If the flow is subcritical, the eminent slopes of the isopycnal surfaces through the Bussol Strait would be explained by the advection–diffusion balance.

An advection–diffusion balance of the density is

$$u\frac{\partial \rho}{\partial x} = \kappa \frac{\partial^2 \rho}{\partial z^2}$$

where u is the horizontal velocity in the along-channel direction, κ is the vertical diffusivity, and vertical velocity is assumed to be very small compared with horizontal velocity. A rough estimate of the vertical diffusivity κ is given by

$$\kappa \sim \frac{\delta \rho}{\Delta \rho}\frac{\Delta z^2}{\delta x}u$$

where δ and Δ denote the horizontal and vertical finite differences, respectively. Typical values of δ and Δ may be given from the density field in Figure 3 ($\Delta z \sim 100$ m, $\delta x \sim 30$ km). If u is chosen to be 1 m s^{-1}, κ becomes 0.3 m^2 s^{-1}. This estimated value appears to be too large, but the strong tidal mixing in the strait might provide such a large mixing.

When the diffusion effect is very large, the dissipation would contribute significantly in the integral (2). Pedlosky et al. (1997) discussed the frictional effect near the strait in the Island Rule (5). If we take Δ as the channel width, δ_M as the diffusive boundary layer width, and α as the ratio of the channel length to the meridional extent of the island, the parameter

$$R_\Delta = \frac{\Delta}{\delta_M \alpha^{1/3}}$$

becomes a determinative parameter. When it is less than 1.5, significant reduction of the transport would occur. For the Bussol Strait, $\Delta \sim 20$ km, $\delta_M \sim 50$ km, and $\alpha \sim 0.1$. So, R_Δ is 0.9, and the reduction due to diffusion would be significant. According to Pedlosky et al. (1997), the transport may become half of that estimated by the Island Rule without diffusion, discussed in the previous section. Then, the transport through the Bussol Strait would be about 5 Sv.

The Island Rule may be modified to be applicable for the hydraulically controlled flow. In the derivation of the Island Rule (2) from (1), it is implicitly assumed that $B = g'D + |\vec{u}|^2/2$ is continuous and is single-valued and that it vanishes when integrated along a closed circuit. Under the hydraulically controlled condition, the flow just downstream of the strait is supercritical, and a hydraulic jump should exist in order that the supercritical flow is connected to the subcritical flow further downstream. The transport and momentum are conserved across the jump, but the energy is dissipated at the jump. If we denote this energy dissipation with $-\delta B$, we have

the modified Island Rule

$$\Phi = \frac{\int_{y_s}^{y_n} \Psi_s(x_+, y)\mathrm{d}y - \delta B/\beta}{y_n - y_s}. \qquad (11)$$

The relative contribution of the hydraulic effect is stronger for shorter islands or for stronger energy dissipation.

Conclusion

In this paper, transport through the Bussol Strait is estimated by using simple models. Both the global constraint (Island Rule) and local constraint (hydraulics) yield estimates of transport from 5 to 10 Sv. The models can be improved if an active lower layer is added, and if a seasonal or shorter variability term is considered. However, much more elaborate observations are needed to model the oceanic state under consideration.

Acknowledgments

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